The Effect of Applied Force on Drainage of the Film Between a Liquid Drop and Horizontal Surface

The effect of applied force, drop volume, and physical properties on the rate of thinning of the draining films above and below a drop trapped between two horizontal surfaces has been examined theoretically and experimentally. The variation in film thickness with position and time has been obtained photographically and from capacitance measurements. The observed rate of film thinning may be predicted by the simple uniform film model with one immobile surface and, surprisingly, decreases when the applied force and drop volume increases.

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SCOPE

Coalescence of liquid drops and gas bubbles with their homophase is determined by the rate of drainage of the intervening fluid film. This increases as the force pressing the surfaces of the film together increases and as the area of the film decreases. If the area of the film is constant, then the rate of drainage will increase with the applied force. However, if the area increases with the applied force, it is not clear what the effect on the rate of drainage will be. Such a situation arises momentarily when two drops collide (Scheele and Lang, 1971) or when a drop impacts a solid surface. It may also occur when the suction in the Plateau border of a gas-liquid foam increases due to a decrease in the interfacial curvature just outside the periphery of the film (Barber, 1973). Forces also arise from the presence of surrounding drops in close packed liquid-liquid dispersions, but these tend to constrain the area of the film to some extent (Hartland, 1970). It is thus important to examine the effect of applied forces on the drainage of thin films because such forces occur in practice and also to extend our basic understanding of the film thinning process which precedes coalescence. This paper examines the effect of a steady applied force on the rate of drainage of the films above and below an unconstrained liquid drop trapped between two horizontal surfaces experimentally and theoretically.

CONCLUSIONS AND SIGNIFICANCE

By using pairs of liquids with close refractive indices, the initial variation in thickness with position and time of the films above and below a drop trapped between two horizontal surfaces has been obtained photographically. At longer drainage times the average rate of film thinning was obtained from capacitance measurements. Both sets of experimental results show that the rate of thinning decreases as the applied force and drop volume increases. The effect of drop volume is not unexpected, but it certainly could not be argued on intuitive grounds that the application of a steady force to a drop or bubble would actually decrease the drainage of the film beneath it and hence the rate of coalescence.

For a uniform film of viscosity μ, the variation in film thickness δ with time t is given by

$$-\frac{d\delta}{dt} = \frac{8\pi}{3n^2}\frac{\delta^3}{\mu}\frac{f}{a^2}$$

so the rate of film thinning is proportional to $f/(a\,n)^2$ where f is the force pressing on the film of area a and n is the number of immobile surfaces bounding the film. The effect of an applied force f on the area a has been previously obtained from the differential equations governing the shape of the drop. The theoretical solution shows that f/a^2 decreases as the applied force, drop volume, and density difference increases and the interfacial tension decreases. Substituting these values of f/a^2 with the experimentally determined rates of film thinning into the above equation enables the variation of n with time to be obtained. This variation shows that the value of n is initially between one and two but approaches one at longer drainage times, which is the expected behavior for the draining film between a liquid-liquid interface and a solid surface. This variation in n and the agreement between the effect of applied force and drop volume on the theoretical values of f/a^2 and the experimentally observed rates of film thinning must be interpreted as providing strong support for the simple uniform film model.

Drops in a close packed dispersion experience forces from the surrounding drops which affect their shape and the drainage of the films of continuous phase which separate them. A previous paper (Wood and Hartland, 1972) discussed the shape of a drop trapped between two horizontal surfaces. The differential equations governing the shape of the surface of the drop were solved

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numerically and the effect of drop volume v and applied force f on the areas a_t and a_b of the upper and lower draining films was obtained. Lengths were made dimensionless using the factor $(\Delta\rho g/\sigma)^{1/2}$ and forces using $(\Delta\rho g)^{1/2}/\sigma^{3/2}$. The theoretical drop dimensions were checked experimentally by applying known forces to a drop and recording the shape photographically.

This paper discusses the effect of an applied force on the rate of thinning of the draining films above and below the drop. Because of the pressure gradient associated with flow in the film and because a fluid-liquid interface must deform when there is a pressure difference across it, the film thickness varies with position, giving rise to the well-known dimple (Frankel and Mysels, 1962; Hartland 1969a). An applied force changes the distribution of pressure in the film and hence the variation in thickness with position and time.

By using pairs of liquids of close refractive indices, the variation in film thickness with position and time may be recorded photographically for short drainage times (Hartland, 1969a). The average film thickness δ is obtained as a function of time by measuring the capacitance C of the film given by

$$C = \epsilon\, \epsilon_0 a/\delta \qquad (1)$$

where ϵ is the dielectric constant of the film and ϵ_0 $(= 8.83 \times 10^{-12}\ \text{F/m})$ is the permeability of free space. The value of a may be obtained photographically or from the known effect of the physical properties, drop volume, and applied force on the drop shape (Wood and Hartland, 1972). As δ decreases, the value of C increases, so the method is particularly sensitive at longer drainage times.

For a uniform film of viscosity μ, the variation in film thickness δ with time t is given by (Reynolds, 1882; Hartland, 1967)

$$-\frac{d\delta}{dt} = \frac{8\pi}{3n^2}\,\frac{\delta^3}{\mu}\,\frac{f}{a^2} \qquad (2)$$

where n is the number of immobile interfaces bounding the film. Knowing δ and $d\delta/dt$ from the capacitance measurements thus enables the variation of n with time to be obtained. The value of n is 2 for two immobile surfaces, 1 for one immobile and one fully mobile surface and 0 for two fully mobile surfaces. Values between 1 and 2 indicate that at least one interface is partially mobile and between 0 and 1 that both interfaces are partially mobile. Equation (2) shows that the rate of film thinning apparently increases with applied force f, but the effect is complicated because the force also increases the area of the film and it is the factor f/a^2 which determines the rate of thinning. It is therefore important to examine the effect of f, drop volume v, and physical properties $\Delta\rho$ and σ on the factor f/a^2 for both the upper and lower draining films.

The force pressing on the bottom film f_b is related to that on the upper film f_t by

$$f_b = f_t + v\Delta\rho g \qquad (3)$$

When $v\Delta\rho g << f_t$ so that $f_b = f_t = f$, the areas of the draining films are equal so that $a_b = a_t = a$ and the surface of the drop is that of a semicircle with radius r given by $\sigma a/f$ (Charles and Mason, 1960). For this limiting case, the volume is given by $v = 2ra = 2\sigma a^2/f$ so that:

$$a_t = a_b = (vf/2\sigma)^{1/2} \qquad (4)$$

$$x_t = x_b = x_e = (vf/2\pi^2\sigma)^{1/4} \qquad (5)$$

and

$$y_b = 2y_e = (2\sigma v/f)^{1/2} \qquad (6)$$

This variation of a_t and a_b, x_t, x_b and x_e, and of y_b and y_e with v and f may be seen in Figures 2 to 8 of Wood and Hartland (1972) when f is large compared with v.

In addition it follows that

$$f/a^2 = 2\sigma/v \qquad (7)$$

which is independent of f and that the average pressure in the draining film is given by

$$f/a = (2\sigma f/v)^{1/2} \qquad (8)$$

EXPERIMENT

The heavy liquid was 41.5% by volume aqueous Lyles Golden Syrup and the light liquid was Silicone Fluid MS 200 manufactured by Midland Silicones Ltd. The physical properties of the liquids measured at 22.5°C are given in Table 1 together with the factors derived from the properties which were used to make the forces, volumes, and areas dimensionless. The addition of ½% by volume aqueous potassium iodide solution saturated at 25°C to the Golden Syrup rendered the drops conducting.

Golden Syrup drops of volume ranging between 0.05 and 0.4 m were trapped between two horizontal stainless steel plates in a glass cell containing the light liquid, and the applied force was measured using a top pan balance as previously described (Wood and Hartland, 1972). A wire was attached to one corner of the stainless steel plate on the bottom of the glass cell and another attached to the syringe needle. The drop of Golden Syrup was introduced from a Hamilton gastight syringe and a needle of 0.01 cm diameter was inserted into

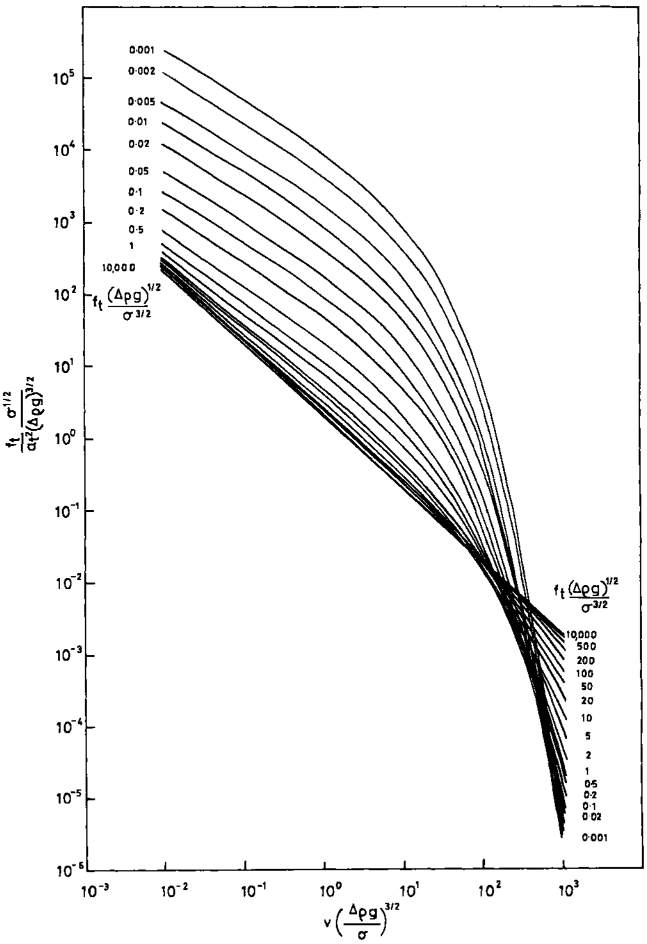


Fig 1. Variation of $(f_t/a_t^2)\ \sigma^{1/2}/(\Delta\rho g)^{3/2}$ (vertical scale) for upper film with drop volume $v(\Delta\rho g/\sigma)^{3/2}$ (horizontal scale) for different values of applied force $f_t(\Delta\rho g)^{1/2}/\sigma^{3/2}$.

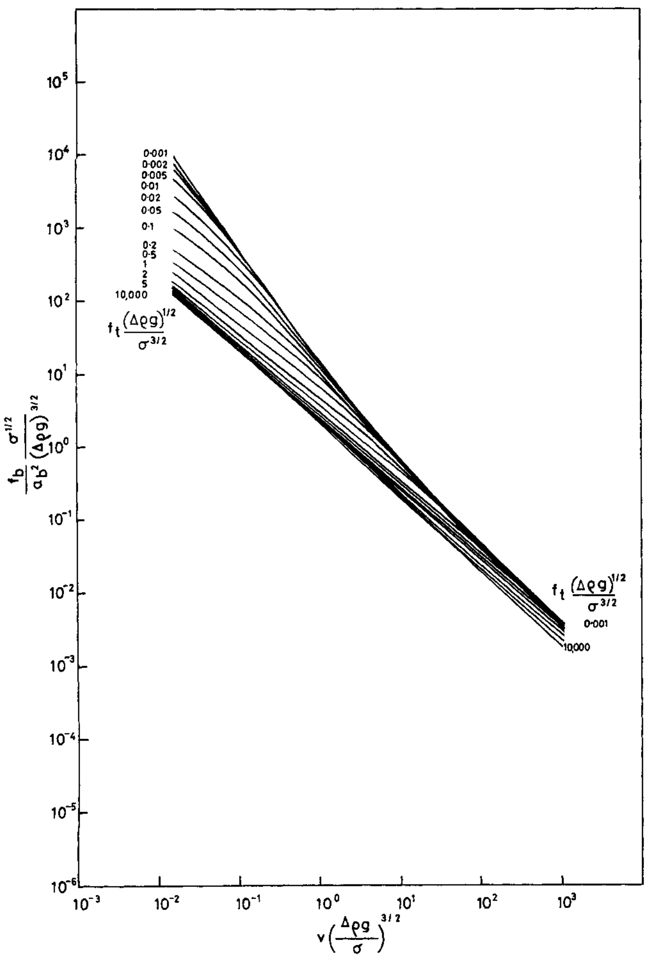


Fig. 2. Variation of $(f_b/a_b^2)\ \sigma^{1/2}/(\Delta\rho g)^{3/2}$ (vertical scale) for lower film with drop volume $v(\Delta\rho g\sigma)^{3/2}$ (horizontal scale) for different values of applied force $f_t(\Delta\rho g)^{1/2}/\sigma^{3/2}$.

TABLE 1. PHYSICAL PROPERTIES OF LIQUIDS USED AT 22.5°C

Physical property	41.5% v/v aqueous golden syrup + 0.5% v/v satd aq KI	Silicone fluid MS 200
Refractive index	1.405	1.404
Dielectric constant	—	2.82
Viscosity, μ g/cm sec	0.592	121.25
Density, ρ g/cm^3	1.187	0.970
Density difference, $\Delta\rho$, g/cm^3	0.217	
Interfacial tension, σ, dynes/cm	37.8	
$(\Delta\rho g/\sigma)^{3/2}$ cm^{-3}	13.5	
$(\Delta\rho g)^{1/2}/\sigma^{3/2}$ dynes^{-1}	0.0630	
$\sigma^{1/2}/(\Delta\rho g)^{3/2}$ dynes^{-1} cm^4	0.00196	
$(\Delta\rho g\sigma)^{-1/2}$ dynes^{-1} cm^2	0.0111	

the side of the drop. These three electrical connections were linked to a Wayne-Kerr Universal Bridge via a two-way switch. The variations with time of the capacitance of the draining films above and below the drop were followed using an autobalance adapter by switching between the top and bottom film circuits. The lead to the bottom plate was independently supported, and the final electrical connection was made with a very fine copper wire spring which did not significantly affect the balance reading. Various applied forces including zero were investigated for each drop volume. Photographs were taken from the side at known times, using back lighting as previously described (Hartland, 1969a), the camera being focused on the mid-plane of the drop so that the draining film and its reflection in the polished stainless steel plate could be seen.

THEORY

The solutions of the differential equations governing the effect of applied force on drop shape give the dimensionless areas of the upper and lower draining films $a_t\Delta\rho g/\sigma$ and $a_b\Delta\rho g/\sigma$ in terms of the dimensionless force $f_t(\Delta\rho g)^{1/2}/\sigma^{3/2}$ applied to the upper plate and dimensionless drop volume $v(\Delta\rho g/\sigma)^{3/2}$ (Wood and Hartland, 1972). Figures 1 and 2 show the variation in dimensionless value of $(f/a^2)\sigma^{1/2}/(\Delta\sigma g)^{3/2}$ for the upper and lower films with the dimensionless force and drop volume calculated from the original data. For the upper film $(f_t/a_t^2)\sigma^{1/2}/(\Delta\rho g)^{3/2}$ decreases with applied force $f_t(\Delta\rho g)^{1/2}/\sigma^{3/2}$ except for large values of $v(\Delta\rho g/\sigma)^{3/2}$. For the lower film $(f_b/a_b^2)\sigma^{1/2}/(\Delta\rho g)^{3/2}$ always decreases with $f_t(\Delta\rho g)^{1/2}/\sigma^{3/2}$ for all values of $v(\Delta\rho g/\sigma)^{3/2}$, but the effect is not marked. This means that for a drop of given volume v and constant physical properties $\Delta\rho$ and σ the values of f_b/a_b^2 will always decrease as f_t increases, and the value of f_t/a_t^2 will decrease as f_t increases, providing v is not large. Equation (2) thus suggests that the rate of thinning of the bottom film will always decrease with the force applied to it and that this is also true for the top film providing the drop volume is not too large.

Figures 1 and 2 also show that for given physical properties $\Delta\rho$ and σ, the value of f/a^2 for the upper and lower

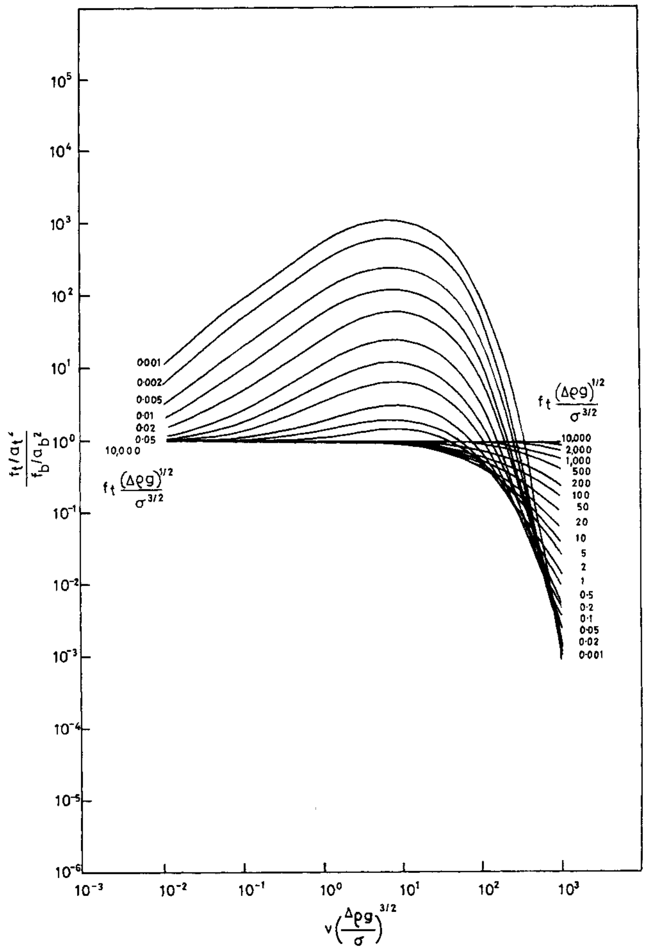


Fig. 3. Variation of ratio $(f_t/a_t^2)/(f_b/a_b^2)$ (vertical scale) for upper and lower films with drop volume $v(\Delta\sigma g/\sigma)^{3/2}$ (horizontal scale) for different values of applied force $f_t(\Delta\rho g)^{1/2}/\sigma^{3/2}$.

Table 2. Dimensionless Groups Used to Investigate the Effect of Applied Force f_t, Drop Volume v, and Physical Properties $\Delta\rho$ and σ on the Drainage Factor f/a^2

Dimensionless f_t	Dimensionless $v, \Delta\rho, \sigma$	Dimensionless f/a^2	Effect	Constant
$f_t \cdot \frac{(\Delta\rho g)^{1/2}}{\sigma^{3/2}}$	$v \cdot \left(\frac{\Delta\rho g}{\sigma}\right)^{3/2}$	$\frac{f}{a^2} \cdot \frac{\sigma^{1/2}}{(\Delta\rho g)^{3/2}}$	f_t and v	$\Delta\rho$ and σ
$f_t \cdot \frac{1}{v^{1/3}\sigma}$	$\Delta\rho \cdot \frac{g v^{2/3}}{\sigma}$	$\frac{f}{a^2} \cdot \frac{v}{\sigma}$	f_t and $\Delta\rho$	v and σ
$f_t \cdot \frac{1}{v\Delta\rho g}$	$\sigma \cdot \frac{1}{\Delta\rho g v^{2/3}}$	$\frac{f}{a^2} \cdot \frac{v^{1/3}}{\Delta\rho g}$	f_t and σ	v and $\Delta\rho$

films always decreases with increasing drop volume for a constant force applied to the film. Equation (2) thus predicts that the films associated with large drops will always drain more slowly than those associated with small drops. The relative magnitude of the value of f/a^2 for the upper and lower films is shown in Figure 3. The value of f_t/a_t^2 is greater than that of f_b/a_b^2, except for large drop volumes suggesting that the upper film will usually drain faster than the lower film. For the limiting case of large forces Equation (7) predicts that

$$\frac{f_t}{a_t^2}\,\frac{\sigma^{1/2}}{(\Delta\rho g)^{3/2}} = \frac{f_b}{a_b^2}\,\frac{\sigma^{1/2}}{(\Delta\rho g)^{3/2}} = \frac{2}{v}\left(\frac{\sigma}{\Delta\rho g}\right)^{3/2} \quad (9)$$

which is observed in Figures 1, 2, and 3 when $f_t(\Delta\rho g)^{1/2}/\sigma^{3/2} > 10^3$.

It is difficult to evaluate the effect of physical properties on the factors f/a^2 from Figures 1 and 2 because they occur in the dimensionless forms of v, f, and f/a^2. However, it is possible to replace $\Delta\rho$ in the dimensionless f and f/a^2 by $\sigma/(gv^{2/3})$ and to rewrite the dimensionless volume $v(\Delta\rho g/\sigma)^{3/2}$ in the form $\Delta\rho g v^{2/3}/\sigma$ to make it linear in $\Delta\rho$. Similarly σ may be replaced by $\Delta\rho g v^{2/3}$ in the dimensionless f and f/a^2 and the parameter $v(\Delta\rho g/\sigma)^{3/2}$ rewritten $\sigma/(\Delta\rho g v^{2/3})$ to make it linear in σ. The forms of the dimensionless groups used to determine the effect of $\Delta\rho$ and σ on f/a^2 are summarized in Table 2. Interpolation is then required because the values of $f_t/(v^{1/3}\sigma)$ and $f_t/(v\Delta\rho g)$ are no longer constant since $v(\Delta\rho g/\sigma)^{3/2}$ varies for each starting value $x_t(\Delta\rho g/\sigma)^{1/2}$ even though $f_t(\Delta\rho g)^{1/2}/\sigma^{3/2}$ is constant.

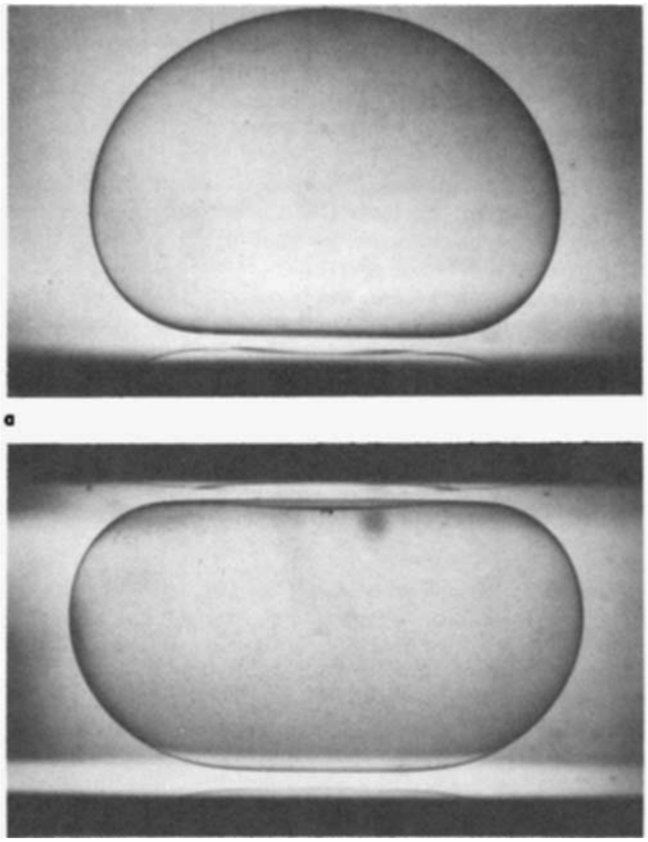
The variation of $(f_t/a_t^2)(v/\sigma)$ and $(f_b/a_b^2)(v/\sigma)$ with $\Delta\rho g v^{2/3}/\sigma$ for different values of $f_t/(v^{1/3}\sigma)$ confirms that there is no dependence of f/a^2 with $\Delta\rho$ when f_t is large. At constant v and σ, the values of f_t/a_t^2 and f_b/a_b^2 decrease with $\Delta\rho$, but the effect is not so marked (providing $\Delta\rho$ is not too large) as that of v at constant $\Delta\rho$ and σ discussed above. The ratio of $(f_t/a_t^2)/(f_b/a_b^2)$ is greater than unity providing $\Delta\rho$ is not too large.

The variation of $(f_t/a_t^2)(v^{1/3}/\Delta\rho g)$ and $(f_b/a_b^2)(v^{1/3}/\Delta\rho g)$ with $\sigma/(\Delta\rho g v^{2/3})$ for different values of $f_t/(v\Delta\rho g)$ shows that at constant v and $\Delta\rho$, the values of f_t/a_t^2 and f_b/a_b^2 increase with σ; for large forces the increase is linear. The ratio $(f_t/a_t^2)/f_b/a_b^2$ is always greater than unity providing σ is not very small.

RESULTS AND DISCUSSION

Photographic Measurements

A typical sequence of photographs showing the profiles of the draining films above and below the drop and their reflection in the polished flat plates is shown in Figures



a

b

Fig. 4. Photographs of a 0.25 ml drop of aqueous golden syrup in silicone oil trapped between horizontal plates: (a) Zero applied force, 60 sec. drainage time; (b) Applied force 12.3 dynes, 65 sec. drainage time.

4 and 5 for a 0.25 ml drop of aqueous golden syrup in silicone oil. In Figures 4a and 4b, the applied forces are 0 and 12.3 dynes and the drainage times 60 and 65 seconds respectively. In Figures 5a to 5d the applied force is 47.3 dynes and the drainage times 45, 90, 240, and 600 seconds. The distance between the profile and its reflection at any point is equal to twice the local film thickness.

The variation in film thickness with position was measured from an enlarged image, the magnification being obtained from the known diameter of the syringe needle. The magnification was usually about 40× and distances could be read to better than 0.05 cm so that actual distances could be measured to about 10^{-3} cm.

Typical variations in thickness with position and time for the films above and below a 0.25 ml drop of aqueous golden syrup in silicone oil are shown in Figures 6a and 6e for three different forces, f_t applied to the upper plate, 0, 12.3, and 47.3 dynes.

Figures 6a to 6e show that the rate of thinning decreases as the force applied to the film increases, and consequently the bottom film thins more slowly than the upper film. This behavior for symmetrical drainage is as predicted by Equation (2) and Figures 1 and 2 which show that f/a^2 decreases with the applied force.

However, for larger forces and larger drops, unsymmetrical drainage may occur which leads to rapid film thinning. Figures 6a to 6e also show that the area of the draining film increases with the applied force, as indicated by the available theoretical solutions for the drop shape. The force f_b applied to the bottom film is greater than f_t by an amount $v\Delta\rho g$ and so the area of the bottom film is correspondingly greater. In addition, the size of the dimple increases with the applied force and so is also greater for the bottom film. This is because the average dynamic pressure in the film f/a always increases with the applied force for a given drop volume v and physical properties $\Delta\rho$ and σ, as shown in Figures 7 and 8 for the upper and lower films. The gradient of the dynamic pressure (measured relative to the hydrostatic pressure) therefore increases with f because the dynamic pressure always falls to zero at the edge of the film (Hartland, 1971). The average excess pressure in the film above that in the drop is zero (Hartland, 1969b), and so the excess pressure in the film at any point is the local value of the dynamic pressure less the average value. An increased force thus increases the dynamic pressure gradient which magnifies the local excess pressure in the film and makes the dimple more pronounced. The profiles in Figures 6a to 6e show that, as expected, the dimple does not collapse as the film thins for the force pressing on the film remains unchanged.

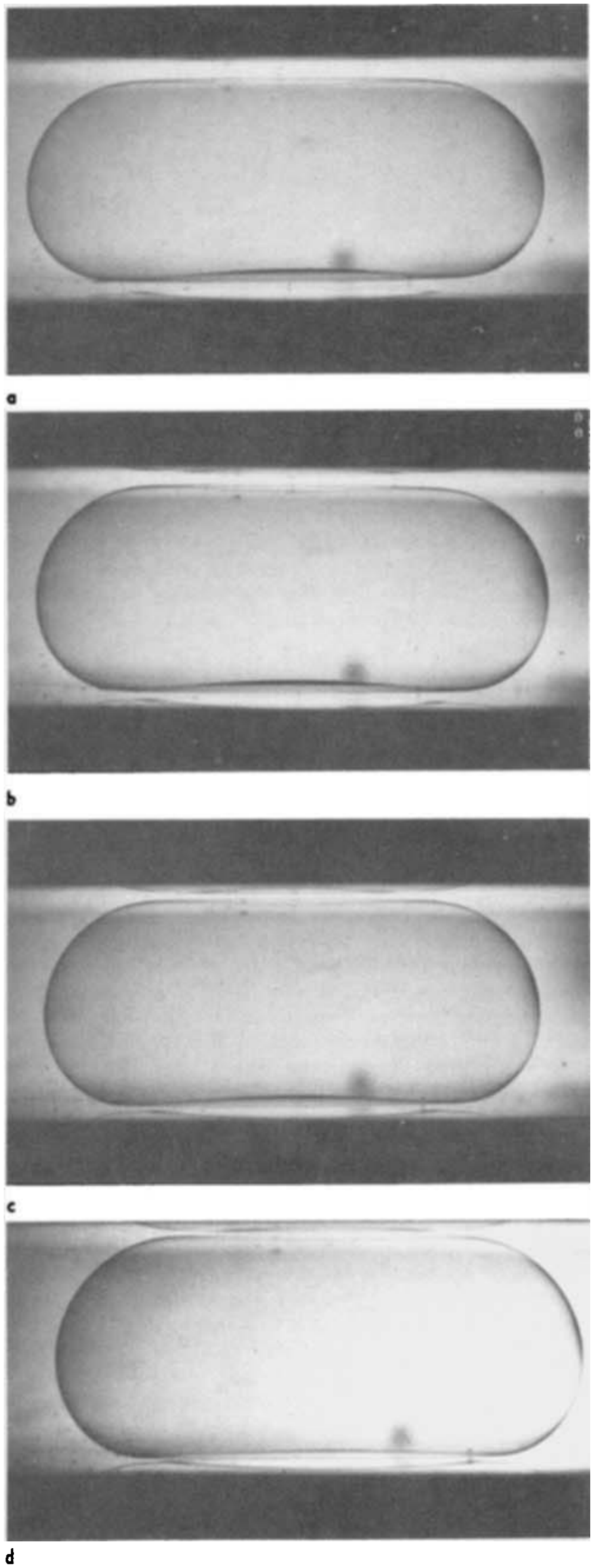
Capacitance Measurements

The variation in δ with time t for the upper and lower films obtained from capacitance measurements using Equation (1) also indicates that the rate of film thinning decreases with applied force and that the upper film thins faster than the lower film. Substituting values of δ and $d\delta/dt$ into Equation (2) together with the theoretical value of f/a^2 obtained from Figures 1 and 2 together with the conversion factors given in Table 1 enables the variation with time of the number n of immobile interfaces bounding the film to be obtained. This is shown in Figure 9 for a 0.15 ml drop of golden syrup in silicone oil for applied forces ranging from 0 to 725 dynes.

Initially the value of n is usually between 1 and 2, falling to about 1 after about 1000 seconds because the mobility of the liquid-liquid interface increases with time. The low initial value of n for the largest force is probably due to unsymmetrical drainage. A value of n equal to 1, corresponding to one immobile and one fully mobile interface is the final value to be expected for a liquid drop having no initial circulation and an uncontaminated surface approaching a solid surface. The observed variation of n with time must therefore be interpreted as providing strong evidence for the uniform film model. Even though the film is known to be dimpled, it behaves as though it were uniform in thickness.

CONCLUSIONS

1. The initial variation in thickness with time and position of the draining films above and below a liquid drop trapped between two horizontal surfaces has been observed photographically using pairs of liquids with close refractive indices. For drops of aqueous golden syrup in silicone oil, both the area of the draining film and the size



a b c d

Fig. 5. Photographs of a 0.25 ml drop of aqueous golden syrup in silicone oil trapped between horizontal plates with an applied force of 47.3 dynes at different drainage times: (a) 45 sec, (b) 90 sec, (c) 240 sec, (d) 600 sec.

of the dimple increase with the magnitude of the force applied to the upper surface for drop volumes between 0.05 and 0.4 ml and forces between 0 and 262.8 dynes. The rate of film thinning decreases as the applied force and drop volume increases and the lower film thins slower than the upper one. For 0.25 ml drops after 1000 seconds the film thickness has decreased to between 10^{-4} and 10^{-5} m and the dimple begins to collapse so the film becomes more uniform in thickness.

2. The variation in average film thickness with time has been obtained from capacitance measurements up to times of 10,000 seconds. The rate of film thinning again decreases with increase of applied force and drop volume.

3. For a uniform film the rate of film thinning is given by

$$-\frac{d\delta}{dt} = \frac{8\pi}{3n^2}\,\frac{\delta^3}{\mu}\,\frac{f}{a^2}$$

where n is the number of immobile interfaces bounding the film. Using the variation of δ with t obtained from capacitance measurements and the known values of f and a enables the variation of n with time to be obtained. For both films, initially the value of n is between 1 and 2, but it decreases with time and approaches the value 1. This is the expected variation for a draining film between a liquid drop and solid surface in which the liquid interface is gradually set in motion by flow in the film. The observed experimental variation of n with time provides strong evidence for the uniform film model of film drainage.

4. Equation (2) shows that the rate of thinning of a uniform film is proportional to f/a^2 and the variation of this factor with the applied force, drop volume, density difference and interfacial tension may be obtained from the known variation in drop shape. For the upper film f_t/a_t^2 always decreases with the drop volume v and also with the applied force f_t providing the drop volume is not too large; for the lower film f_b/a_b^2 always decreases with v and f_t. For both films, f/a^2 decreases as the density difference $\Delta\rho$ increases and interfacial tension σ decreases. The ratio of $(f_t/a_t^2)/(f_b/a_b^2)$ is greater than unity pro-

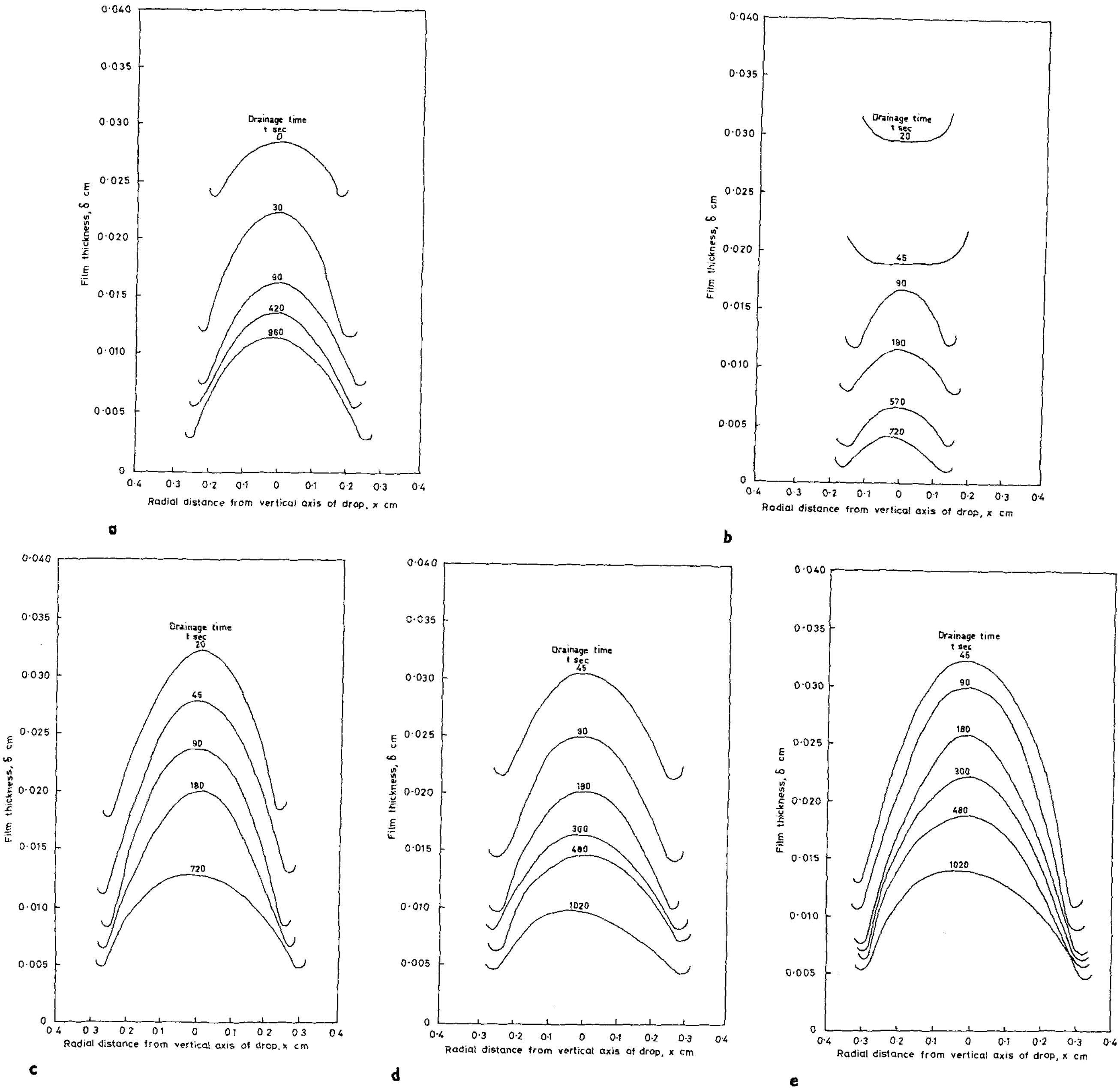


Fig. 6. Variation in thickness δ with position x and time t for the films above and below a 0.25 ml drop of aqueous golden syrup in silicone oil with different applied forces: (a) lower film 0 dyne, (b) upper film 12.3 dynes, (c) lower film 12.3 dynes, (d) upper film 47.3 dynes, (e) lower film 47.3 dynes.

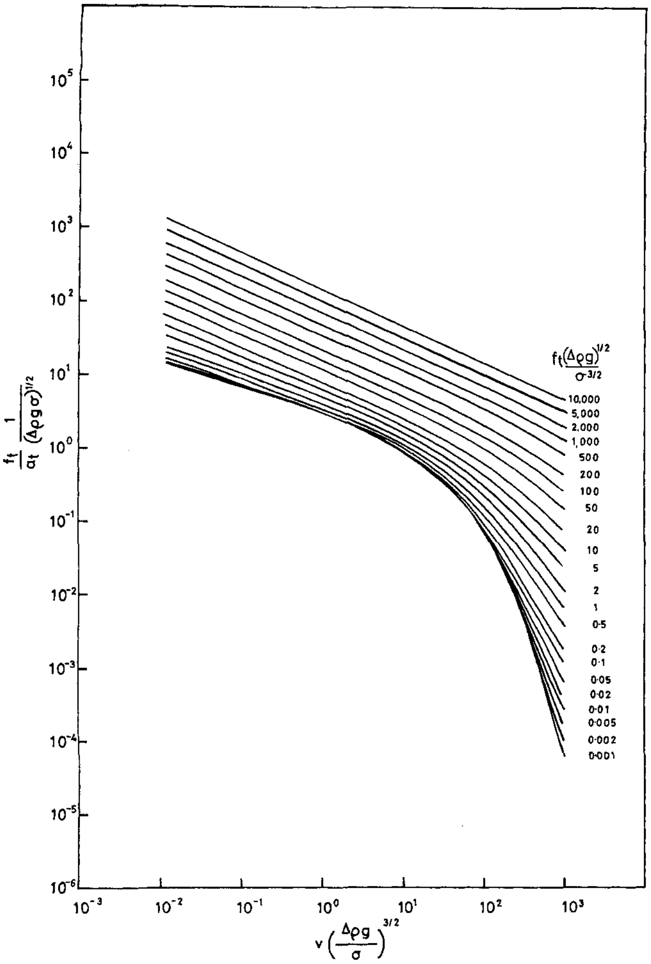


Fig. 7. Variation in $(f_t/a_t)/(\Delta\rho g\sigma)^{1/2}$ (vertical scale) for upper film with drop volume $v(\Delta\rho g/\sigma)^{3/2}$ (horizontal scale) for different values of applied force $f_t(\Delta\rho g)^{1/2}/\sigma^{3/2}$.

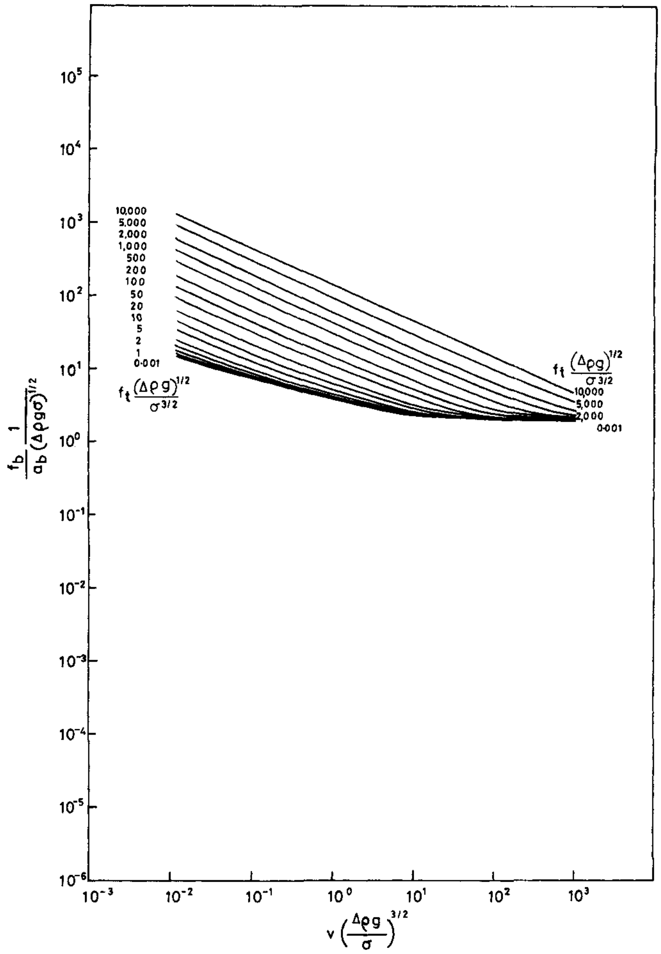


Fig. 8. Variation in $(f_b/a_b)/(\Delta\rho g\sigma)^{1/2}$ (vertical scale) for lower film with drop volume $v(\Delta\rho g/\sigma)^{3/2}$ (horizontal scale) for different values of applied force $f_t(\Delta\rho g)^{1/2}/\sigma^{3/2}$.

viding that v is not too large, suggesting that the upper film thins faster than the lower film.

The predicted effect of applied force f_t and drop volume v on the rate of film thinning $-d\delta/dt$ agrees with that experimentally observed for the upper and lower films.

5. The theoretical variation of drop shape also predicts that the average pressure in the film, f/a increases with the applied force and decreases with drop volume. It follows that the size of the dimple should increase with f_t and decrease with v, as confirmed by experiment.

NOTATION

- a = area of draining film
- C = electrical capacitance of draining film
- f = force acting on plate
- g = acceleration due to gravity
- n = number of immobile interfaces
- t = drainage time
- v = drop volume
- x = radial distance from vertical axis of drop

Greek Symbols

- δ = film thickness
- $\Delta\rho$ = density difference between heavy and light phases
- μ = viscosity
- σ = interfacial tension
- ϵ_o = permittivity of free space, 8.83×10^{-12} F/m
- ϵ = dielectric constant

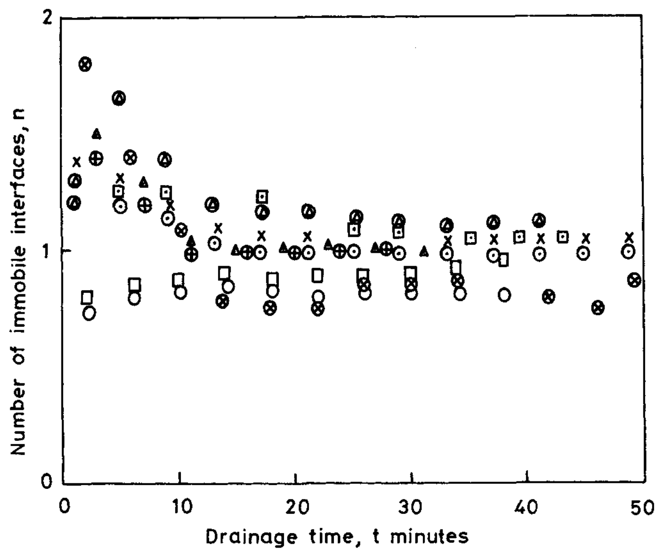


Fig. 9. Variation with time of the number of immobile interfaces n bounding the draining films above and below a 0.15 ml drop of golden syrup in silicone oil trapped between horizontal plates for different applied forces.

Applied Force, dynes	0	11·9	20·3	31·9	72·4
Top Film		⊙	⊗ (shaded)	⊕	□
Bottom Film	⊗	x	⊡	▲	○

Subscripts

b = refers to bottom plate
t = refers to top plate

LITERATURE CITED

Barber, A. D., "A Model for a Cellular Foam," Ph.D. thesis, Univ. Nottingham, England (1973).

Charles, G. E., and S. G. Mason, "Coalescence of Liquid Drops with Flat Liquid Liquid Interfaces," *J. Colloid Sci.*, **15**, 236 (1960).

Frankel, S., and K. J. Mysels, "On The Dimpling during the Approach of Two Interfaces," *J. Phys. Chem.*, **66**, 190 (1962).

Hartland, S., "The Approach of a Liquid Drop to a Flat Plate," *Chem. Eng. Sci.*, **22**, 1675 (1967).

———., "The Profile of the Draining Film Beneath a Liquid Drop Approaching a Plane Interface," *Chem. Eng. Progr. Symp. Ser. No. 91*, **65**, 82 (1969a).

———., "The Shape of a Fluid Drop Approaching an Interface," *Can. J. Chem. Eng.*, **47**, 221 (1969b).

———., "Unsymmetrical Drainage Beneath a Liquid Drop Approaching an Inclined Plane," *Chem. Eng. J.*, **1**, 258 (1970).

———., "The Pressure Distribution in Axisymmetric Draining Films," *J. Coll. Interface Science*, **35**, 227 (1971).

Reynolds, O. "On The Theory of Lubrication," *Phil. Trans. R. Soc. London*, **A177**, 157 (1886).

Scheele, G. F., and D. E. Lang, "An Experimental Study of Factors which Promote Coalescence of Two Colliding Drops Suspended in Water—I," *Chem. Eng. Sci.*, **26**, 1867 (1971).

Wood, S. M., and S. Hartland, "The Shape of a Drop Trapped Between Two Horizontal Surfaces," *AIChE J.*, **18**, 1041 (1972).

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Interfacial Areas of Liquid-Liquid Dispersions from Light Transmission Measurements

A mathematical model simulating light transmission through liquid-liquid dispersions has been developed. Numerical solutions of the model for various drop size distributions show that the fraction of parallel light which passes through a dispersion is a unique function of a dimensionless group, here named the Transmission Number, regardless of the drop size distribution. The results, which were verified by actual light transmission experiments, show that the interfacial area of a liquid-liquid dispersion can be calculated from a light reading provided that the light detector receives only parallel light. Application of the results to other two-phase dispersions is indicated.

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SCOPE

When two immiscible liquids are brought into contact to form a dispersion, knowledge of the interfacial area is frequently important. For example, in the analysis of heat or mass transfer between two phases, the interfacial area must be known in order to determine the corresponding transfer coefficient. If the interfacial area can be measured, then the area-free coefficient can be found, giving information on the independent effects of area and resistance. In this way, direct measurement of interfacial area can contribute to a better understanding of heat and mass transfer and their industrial applications.

There are cases where drop size itself is an important consideration, particularly in emulsion and other types of polymerization. The same data from interfacial area measurements will also give average drop size in terms of Sauter mean diameter.

Several methods have been used to measure the interfacial area or drop size of dispersions, but none to date seem to combine simplicity with accuracy. Photographic techniques are probably the most accurate method of measuring interfacial area, but the procedure usually requires many pictures and lengthy times for analysis. Furthermore, in a concentrated dispersion, only the drops nearest the camera can be measured. Light transmission through dispersions has proven to be the simplest method, but the theory, until now, has often been limited to dilute dispersions or to a specific detector system, as in the works of Bailey (1946), Langlois et al. (1954), Vermeulen et al. (1955), and Trice and Rodger (1956).

The most widely used theory for light transmission through liquid-liquid dispersions has been developed by Calderbank (1958) where a uniform drop size was assumed. Excellent summaries of the previous work in this field are given by Trice and Rodger (1956) and Calderbank (1958).

In this investigation, light transmission theory has been applied to nonuniform dispersions to determine the rela-

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